

A COMPARISON OF HOMOGENEOUS STELLAR MODELS BASED ON THE COX-STEWART AND CARSON OPACITIES

RICHARD STOTHERS

Institute for Space Studies, Goddard Space Flight Center, NASA

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ABSTRACT

Homogeneous stellar models in the mass range $1\text{--}120 M_{\odot}$ have been constructed for two Population I chemical compositions by employing (a) the Cox-Stewart "hydrogenic" opacities and (b) the new Carson "Thomas-Fermi" opacities. The differences between the models at fixed mass and chemical composition are typically found to be (except at the highest masses) as small as the differences that are caused by the choice of linear interpolation, quadratic interpolation, or a fitted formula in the opacity tables.

Subject headings: interiors, stellar — opacities

I. INTRODUCTION

Radiative opacity is a still uncertain quantity in stellar models. Keller and Meyerott (1955) and Cox and Stewart (1965) have produced the main body of modern opacities based on the "hydrogenic" model of the atom, while Carson, Mayers, and Stibbs (1968) and Carson (1974) have produced opacities based on the hot "Thomas-Fermi" model of the atom for all elements heavier than hydrogen and helium. Carson has pointed out that these two atomic models may be expected to provide lower and upper limits, respectively, to the "true" opacity. A comparison between stellar models built with the Keller-Meyerott opacities and preliminary Cox-Stewart opacities has already been made by Iben (1963), and has been followed by Cesarsky's (1969) comparison of models constructed with strictly Cox-Stewart opacities with and without line absorption. The differences found in both investigations were small. Rood (1971), however, has shown that comparable differences may arise simply from switching from linear to multipoint interpolation in the opacity tables. Because of the small number of opacity points published by Carson *et al.* (1968), no systematic comparison of models built with their opacities and those of Cox and Stewart has been made, but the limited comparisons available (Novotny 1972; Robertson 1972; Stothers and Ezer 1973; Stothers 1973) indicate large differences between the models. These differences are attributable both to the greater magnitude and to the greater temperature-sensitivity of the Carson-Mayers-Stibbs opacities.

Carson's (1974) new opacities are a considerable improvement over his earlier calculations, and, except at the extremes of high and low density, are now found to be closer to the opacities of Cox and Stewart. Various refinements applied to the Cox-Stewart opacities since 1965 (Cox and Stewart 1970; Merts and Magee 1972) are rather minor and will be ignored here. The present paper makes a systematic comparison of homogeneous stellar models constructed with the currently available "hydrogenic" and "Thomas-Fermi" opacities. It also systematically assesses the

differences among models arising from three different schemes of interpolation in the opacity tables.

II. HOMOGENEOUS STELLAR MODELS

The basic input physics, other than opacity, used in constructing the present models is the same as that used by Stothers (1972). A formula fitted to the Cox-Stewart opacities (Christy 1966; Stothers and Simon 1970) is used as one of the opacity representations. Tabular opacities (in the form $\log \kappa$) are interpolated in the usual quantities $\log T$, $\log \rho$, $\log Z$, and X . We shall adopt the abbreviations: CS, Cox-Stewart; C, Carson; F , fitted formula; L , linear interpolation in the tables; and Q , quadratic interpolation in the tables. The five cases considered in this paper are: CS(F), CS(L), CS(Q), C(L), and C(Q).

The results for homogeneous main-sequence stars with masses in the range $2\text{--}120 M_{\odot}$ are presented in table 1 for a (hydrogen, metals) abundance of $(X, Z) = (0.739, 0.021)$. The models for the case CS(F) in the mass range $5\text{--}120 M_{\odot}$ have already been published by Stothers (1972), and similar models for the case CS(L) in the mass range $2\text{--}100 M_{\odot}$ have been published by Ezer and Cameron (1967). All of these models have convective cores and radiative envelopes. However, a thin convection zone appears near the surface in some of the models, particularly those with "Thomas-Fermi" opacities and with very low and very high masses. In all cases, convection was treated as being adiabatic. The core structures of the models are found to be virtually independent of the opacity representation.

Since models for $1 M_{\odot}$ are qualitatively different from those for higher masses, we have considered them separately. These models have radiative cores, convective envelopes, and the pp chain (rather than the CN cycle) as their main nuclear energy source. To compute them, we have used essentially the same program and input physics as was used by Ezer and Cameron (1965) in obtaining their zero-age main-sequence model for the same chemical composition. The ratio of convective mixing length to pressure scale

TABLE 1
THEORETICAL MODELS FOR HOMOGENEOUS MAIN-SEQUENCE STARS WITH $(X, Z) = (0.739, 0.021)$

CASE	M/M_{\odot}								
	2	3	5	7	10	15	30	60	120
CS(Q):									
$\log L/L_{\odot}$	1.212	1.880	2.688	3.195	3.705	4.240	5.040	5.688	6.218
$\log T_e$	3.985	4.098	4.233	4.318	4.400	4.476	4.598	4.681	4.733
C(Q):									
$\log L/L_{\odot}$	1.202	1.877	2.689	3.198	3.709	4.245	5.042	5.684	6.212
$\log T_e$	3.986	4.100	4.233	4.313	4.390	4.456	4.563	4.632	4.673
C(Q) - CS(Q):									
$\delta \log L/L_{\odot}$	-0.010	-0.003	+0.001	+0.003	+0.004	+0.005	+0.002	-0.004	-0.006
$\delta \log T_e$	+0.001	+0.002	+0.000	-0.005	-0.010	-0.020	-0.035	-0.049	-0.060
C(L) - CS(L):									
$\delta \log L/L_{\odot}$	-0.016	-0.007	+0.002	+0.005	+0.006	+0.006	+0.002	-0.004	-0.010
$\delta \log T_e$	+0.001	+0.001	-0.001	-0.005	-0.010	-0.019	-0.038	-0.055	-0.065
CS(L) - CS(Q):									
$\delta \log L/L_{\odot}$	-0.020	-0.019	-0.019	-0.021	-0.021	-0.018	-0.016	-0.013	-0.006
$\delta \log T_e$	-0.005	-0.005	-0.005	-0.007	-0.007	-0.008	-0.009	-0.008	-0.009
CS(F) - CS(Q):									
$\delta \log L/L_{\odot}$	-0.063	-0.004	+0.031	+0.035	+0.031	+0.021	+0.007	-0.001	-0.005
$\delta \log T_e$	+0.001	+0.011	+0.017	+0.015	+0.012	+0.008	+0.002	-0.001	-0.002

height is taken to be $\alpha = 2$, but the resulting radii and effective temperatures of the models are reliable only for comparative purposes because they are easily altered by a modest change in the uncertain parameter α . We find for the differences between the models, in the sense $C(L) - CS(L)$:

$$\delta \log L/L_{\odot} = -0.090, \quad \delta \log T_e = -0.042.$$

The differences among the models for a given mass are surprisingly small. In fact, typical differences between models constructed with the "hydrogenic" and "Thomas-Fermi" opacities are so slight as to be merely comparable with errors arising from the interpolation scheme itself. The reason is that the two atomic models give similar opacities at moderate densities (low masses), while at low densities (high masses), where the two atomic opacities differ most, electron scattering dominates as the main opacity source. However, for very high masses, the contribution from atomic absorption, though confined to the outer envelope layers, has a large influence on the stellar radius and effective temperature (cf. Stothers

and Chin 1968). Linear interpolation in the opacity tables tends to give larger values of opacity than does multipoint interpolation (Rood 1971)—an effect which shows up in the stellar models in the form of fainter luminosities, larger radii, and cooler effective temperatures (cf. Schwarzschild 1958). Finally, the fitted opacity formula, which was not designed specifically to apply at high densities, seems to work quite well even at the smallest masses considered.

Strictly speaking, the Carson and Cox-Stewart opacities are not properly comparable because the elements within Z and their relative abundances differ somewhat in the two cases. However, a special comparison made by Carson (1974) for an identical chemical composition and for solar conditions of temperature and density indicates that the similarity of opacities found above persists despite a change of elements within Z . One parameter that can be tested here is the total value of Z . The results for four stellar masses are shown in table 2 for the case $Z = 0.044$. It is apparent that the degree of difference between models constructed with "hydrogenic" and "Thomas-Fermi" opacities is only weakly dependent on Z .

TABLE 2
THEORETICAL MODELS FOR HOMOGENEOUS MAIN-SEQUENCE STARS
WITH $(X, Z) = (0.602, 0.044)$

CASE	M/M_{\odot}			
	2	5	15	120
CS(Q):				
$\log L/L_{\odot}$	1.365	2.869	4.409	6.298
$\log T_e$	3.993	4.253	4.496	4.733
C(Q):				
$\log L/L_{\odot}$	1.340	2.839	4.416	6.299
$\log T_e$	3.987	4.240	4.462	4.597
C(Q) - CS(Q):				
$\delta \log L/L_{\odot}$	-0.025	-0.030	+0.007	+0.001
$\delta \log T_e$	-0.006	-0.013	-0.034	-0.136

III. CONCLUSION

Carson's (1974) latest opacities based on the hot Thomas-Fermi model of the atom and the opacities of Cox and Stewart (1965) based on the hydrogenic model have been used in the construction of homogeneous stellar models for two Population I chemical compositions. The differences between the stellar models at fixed mass and chemical composition are typically found to be (except at the highest masses) as small as the differences that are caused by the choice of linear interpolation, quadratic interpolation, or a

fitted formula in the opacity tables. If the present "hydrogenic" and "Thomas-Fermi" opacities do in fact bracket the "true" opacity, any significant remaining uncertainty in the structure of ordinary homogeneous main-sequence stars is not expected to be due to opacity errors.

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RICHARD STOTHERS: Institute for Space Studies, Goddard Space Flight Center, NASA, 2880 Broadway, New York, NY 10025